Solutions to Probe 2012:

**Part I**
1. 8:15 pm
2. 120
3. \( \frac{12}{21} \)
4. 2.22 miles
5. \( \frac{10}{c} \)
6. 8
7. 9

**Part II**
1. \( \frac{300}{7} \) miles/hr ≈ 42.8 \( \frac{mi}{hr} \)
2. 8 diamonds
3. 34 triangles
1. Two cuckoo clocks were brought to the Tick Tock Clock Shop for repairs. Both clocks always told the correct time. However, one cuckoo came out every 45 minutes and the other came out every 55 minutes. One day, both cuckoos came out at 12:00 noon. When was the next time both cuckoos came out together?

Answer: \(8:15\) pm

2. There are just enough microscopes, test tubes and calculators so that every 3 students have to share a microscope, every 4 students have to share a test tube and every 5 students had to share a calculator. The sum of microscopes, test tubes and calculators is 94. How many students are there?

Answer: 120

3. A 6 sided die having its side numbered from 1 till 6 is tricked such that the probability of appearance of each side is proportional to its number. What is the probability of the event “Even”?

Answer: \(\frac{12}{21}\)

4. In a ten-mile race, First beats Second by 1 mile and First beats Third by 3 miles. If the runners maintain constant speed throughout the race, by how many miles does Second beat Third?

Answer: 2.22 miles

5. The sum of the reciprocals of the roots of the equation \(ax^2 + bx + c = 0\) is

Answer: \(-\frac{b}{c}\)

6. In the base ten number system the number 526 means \(5 \cdot 10^2 + 2 \cdot 10 + 6\). In FantasiaLand, however, numbers are written in the base \(r\). Jones purchases an automobile for 440 monetary units (abbreviated m.u.). He gives the salesman a 1000 m.u. bill, and receives, in change, 340 m.u. The base \(r\) is

Answer: 8

7. One side of a given triangle is 18 inches. Inside the triangle a line segment is drawn parallel to this side forming a trapezoid whose area is three-fourth of that of the triangle. The length of this segment, in inches, is:

Answer: 9
1. 1st every 45 min.
   2nd every 55 min., need \( \text{lcm} = \text{least common multiple} \)
   \( \text{lcm}(45, 55) = 5 \times \text{lcm}(9, 11) = 5 \times 99 = 495 \)
   \( \Rightarrow \) after 495 min, both will come out again
   \( 495 = 480 + 15 = 60 \times 8 + 15, \) so 8:15 pm

2. \( m + t + c = 94 \)
   \( s = 3m = 4t = 5c \) \( \Rightarrow m = \frac{s}{3}, t = \frac{s}{4}, c = \frac{s}{5} \)
   \( \Rightarrow \frac{s}{3} + \frac{s}{4} + \frac{s}{5} = 94 \)
   \( \Rightarrow \frac{5s}{60} + \frac{4s}{60} + \frac{5s}{60} = 94 \)
   \( \Rightarrow s \times \frac{14s}{60} = 94 \times 60 \)
   \( \Rightarrow s = \frac{5640}{14} = 400 \)
   \( \Rightarrow s = 400 \)

3. total # of dots = 1 + 2 + 3... + 6 = \( \frac{7 \times 6}{2} = 21 \)
   \( P(\text{"1"}) = \frac{1}{21}, \ P(\text{"2"}) = \frac{2}{21} \) etc
   \( P(\text{"2" or "4" or "6"}) = \frac{2}{21} + \frac{4}{21} + \frac{6}{21} = \frac{12}{21} \)
\( s = \text{distance} (=10) \)
\( a = \text{speed of 1st runner} \)
\( b = \text{speed of 2nd runner} \)
\( c = \text{speed of 3rd runner} \)

\[
10 = a \cdot t_1 \quad s = a \cdot t_1 \quad \frac{s}{t_1} - \frac{1}{t_1} = b \\
q = b \cdot t_1 \quad s-1 = b \cdot t_1' \quad \frac{s-1}{t_1'} - \frac{2}{t_1} = b \\
7 = c \cdot t_1 \quad s-3 = c \cdot t_1' \quad \frac{s-3}{t_1'} - \frac{2}{t_1} = c \\
\]

2nd runner will finish 1 mile in \( b \text{ time} \) \( \frac{s-1}{t_1 + \frac{2}{t_1}} \).

Note that the 2nd will advance his lead over 3rd by

\[
\frac{1}{b} (b-c) = \frac{1}{b} \left( -\frac{1}{t_1} + \frac{3}{t_1} \right) \text{ miles} \\
\]
but \( t_1 = \frac{s-1}{b} \)

So, the overall lead of 2nd over 3rd at the time 2nd finishes is

\[
2 + \frac{1}{b} (b-c) = 2 + \frac{1}{b} \left( -\frac{b}{s-1} + \frac{3b}{s-1} \right) \\
= 2 + \frac{1}{b} \frac{2b}{s-1} = 2 + \frac{2}{s-1} = 2 + \frac{2}{9} = \frac{20}{9} \approx 2.22
\]
\[ \frac{1}{4}A \]

\[ \frac{18 \cdot h}{2} = A \]

\[ \frac{l \cdot s}{2} = \frac{1}{4}A = \frac{1}{4} \cdot \frac{18h}{2} = \frac{9h}{4} \Rightarrow \frac{h}{s} = \frac{48}{2 \cdot 9} = \frac{8}{9} \]

**Geometry**

\[ \frac{h}{s} = \frac{18}{l} \]

So

\[ \frac{h}{s} = \frac{18}{l} = \frac{2l}{9} \]

\[ \Rightarrow \frac{18 \cdot 9}{2} = l^2 \]

\[ 9 = l^2 \]

\[ l = 3 \]
1. One hour out of the station, the locomotive of a freight train develops trouble that slows its speed to $\frac{3}{5}$ of its average speed up to the time of the failure. Continuing at this reduced speed, it reaches its destination two hours late. Had the trouble occurred 50 miles beyond, the delay would have been reduced by 40 minutes. What was the average speed of the train before the locomotive got into trouble?

2. Imagine the symbols below to sit on scales. Scale 1 and 2 are perfectly balanced.
   1. $\Diamond \heartsuit \leftrightarrow \spadesuit \spadesuit \spadesuit \spadesuit$
   2. $\spadesuit \spadesuit \Diamond \Diamond \heartsuit \leftrightarrow \heartsuit \heartsuit$
   3. $\heartsuit \spadesuit \spadesuit \leftrightarrow \ ?$

   How many $\Diamond$'s are needed to balance scale 3?

3. How many triangles?
Part II

1. distance in miles

\[ d \]

\[ \text{slopes} \]

\[ \text{slope } \frac{3}{5} \]

\[ \rightarrow t \text{ in min} \]

\[ 50 \]

\[ \frac{120}{2} \text{mi} \]

\[ s = \text{speed originally} \]

\[ \frac{3}{5} \cdot s \]

\[ d = s \cdot t \]

\[ \text{fill in:} \]

\[ \Delta t: \]

\[ \frac{3}{5} s = \frac{s \cdot x}{80+x} \implies \frac{3}{5} (80+x) = x \]

\[ \frac{240}{5} + \frac{3}{5} x = x \]

\[ 48 = \frac{3}{5} x \]

\[ 24 = \frac{1}{5} x \]

\[ x = 120 \]
\[
\frac{5 \cdot x + 50}{50 + 10 \cdot x + 120} = \frac{3}{5}S
\]

\[x = 120\]

\[
\frac{120S + 50}{50S + 240} = \frac{3}{5}S
\]

\[
\frac{120 + \frac{50}{S}}{240 + \frac{50}{S}} = \frac{3}{5}S
\]

\[120 + \frac{50}{S} = \frac{3}{5} \left(240 + \frac{50}{S}\right) = 3 \left(48 + \frac{10}{S}\right) = 144 + \frac{30}{S}\]

\[\frac{20}{S} = 24\]

\[S - \frac{20}{24} = \frac{10}{14} = \frac{5}{7}\text{ miles/min}\

\[= \frac{5}{7} \text{ miles} = \frac{60.5}{7} \text{ m/hr} = \frac{300}{7} \text{ m/hr} = \frac{12\frac{1}{7}}{7} \text{ m/hr}\]
Part II

2: \[1d + 1h = 4s\]
\[2d + 1h + 2s = 2h\]

\[\begin{align*}
1d + 1h - 4s &= 0 \\
2d - 1h + 2s &= 0
\end{align*}\]

\[\frac{3d}{3d} - \frac{2s}{2s} = 0\]
\[3d = 2s\]
\[s = \frac{3}{2}d\]

\[h + 2s = ?\]
\[= \frac{4s - d}{4} + 2s\]
\[= 6s - d\]
\[= 6 \cdot \frac{3}{2}d - d\]
\[= 9d - d = 8d\]
3:

Proceed in lexicographic order:

123  234  346  468  567  679
125  235  356  469  569  689
126  239  358  479  589
129  246  359
134  247  368
136  249  369
139  256
146  267
149  269
156
159
11

Total:

\[
\begin{array}{cccc}
11 \\
9 \\
6 \\
3 \\
3 \\
4 \\
\end{array}
\]